An optimal control problem arising from a dengue disease transmission model

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\textbf{Article info}

\textbf{Article history:}
Received 11 April 2011
Received in revised form 30 August 2012
Accepted 26 November 2012
Available online 27 December 2012

\textbf{Keywords:}
Dengue disease transmission
Basic reproductive ratio
Optimal control problem

\textbf{Abstract}

An optimal control problem for a host-vector Dengue transmission model is discussed here. In the model, treatments with mosquito repellent are given to adults and children and those who undergo treatment are classified in treated compartments. With this classification, the model consists of 11 dynamic equations. The basic reproductive ratio that represents the epidemic indicator is obtained from the largest eigenvalue of the next generation matrix. The optimal control problem is designed with four control parameters, namely the treatment rates for children and adult compartments, and the drop-out rates from both compartments. The cost functional accounts for the total number of the infected persons, the cost of the treatment, and the cost related to reducing the drop-out rates. Numerical results for the optimal controls and the related dynamics are shown for the case of epidemic prevention and outbreak reduction strategies.
and infectious adults $x_2$, treated infected and infectious adults $x_4$ and recovered human $x_5$. The vector population is divided into two compartments, susceptible vectors $v_1$ and infectious vectors $v_2$.

Let $x = (x_1, x_2, x_3, x_4, x_5, x_6, v_1, v_2)^T \in \mathbb{R}^7$. Transmission diagrams between compartments are shown in Fig. 1. We consider the following dynamical system

$$\frac{dx}{dt} = M(t,x)$$

for the sake of shorter notation that we use. The complete dynamical system is given by

$$(2a) \quad \frac{dx_1}{dt} = A - ax_1 - \theta_1(t)x_1 - b_{ns}x_1v_2 - \mu_1x_1 + (\delta_0 + \delta_1(t)x_2),$$

$$(2b) \quad \frac{dx_2}{dt} = \theta_1(t)x_1 - ax_2 - \mu_2x_2 - (\delta_0 + \delta_1(t)x_2),$$

$$(2c) \quad \frac{dx_3}{dt} = ax_1 - \theta_2(t)x_3 - b_{ns}x_3v_2 - \mu_3x_3 + (\delta_0 + \delta_2(t)x_4),$$

$$(2d) \quad \frac{dx_4}{dt} = ax_2 + \theta_2(t)x_3 - \mu_4x_4 - (\delta_0 + \delta_2(t)x_4),$$

$$(2e) \quad \frac{dx_5}{dt} = \theta_2(t)x_4 - ax_5 - \gamma x_5 - \mu_5x_5 + (\delta_0 + \delta_1(t)x_6),$$

$$(2f) \quad \frac{dx_6}{dt} = \theta_2(t)x_5 - ax_6 - \gamma x_6 - \mu_6x_6 - (\delta_0 + \delta_1(t)x_6),$$

$$(2g) \quad \frac{dx_7}{dt} = \frac{\beta_1x_6v_2}{N_v} + ax_7 - \gamma x_7 - \mu_7x_7 + (\delta_0 + \delta_2(t)x_8),$$

$$(2h) \quad \frac{dx_8}{dt} = \theta_2(t)x_8 + ax_8 + \gamma x_8 - \mu_8x_8 - (\delta_0 + \delta_2(t)x_8),$$

$$(2i) \quad \frac{dx_9}{dt} = \gamma (x_5 + x_6 + x_7 + x_8) - \mu_9x_9,$$

$$(2j) \quad \frac{dv_1}{dt} = B - \frac{b_{ns}v_1(x_5 + x_6)}{N_v} - \mu_1v_1,$$

$$(2k) \quad \frac{dv_2}{dt} = \frac{b_{ns}v_1(x_5 + x_6)}{N_v} - \mu_2v_2,$$

with initial conditions $x(0) = x_0$. For the sake of notation, we use $x_{10} = v_1$ and $x_{11} = v_2$. Note that the removal rate $\delta_0 + \delta_1(t)$ is the sum of the return rate from treated to untreated compartment $\delta_0$ (inverse of the implementation period of repellent to each treated individual) and the drop-out rate $\delta_1(t)$ of treated persons who do not succeed to complete the treatment. The drop-out rate $\delta_1(t)$ depends on the effort being spent for socialization and campaign of the program. For the overall human population $N_e = \sum_{i=1}^{9} x_i$, it holds that

$$\frac{dN_e}{dt} = A - \mu_e N_e$$

and for the total vector population $N_v = v_1 + v_2$ we have

$$\frac{dN_v}{dt} = B - \mu_v N_v.$$  

In general, the natural death rate depends on ages (see [15] for related PDE model with age dependent). In this model, we only use the average natural death rate which is commonly used both for children and adult compartments. Assuming that human and mosquito population to be in equilibria, we obtain $N_e = \frac{A}{\mu_e}$ and $N_v = \frac{B}{\mu_v}$. With this assumption, we could scale the human subpopulations by $\bar{N}_e$ and the vector subpopulations by $\bar{N}_v$. Let $x_i = x_i/\bar{N}_e$ for $i = 1, \ldots, 9$ and $v_j = v_j/\bar{N}_v$ for $j = 1, 2$. We could obtain the normalized system of (2) by replacing $x_i$ with $\bar{x}_i$ with $\bar{v}_j$ and the corresponding parameters $A, B, \beta_i$ with $\bar{A}, \bar{B} = \frac{A}{\bar{N}_e}, \bar{\beta}_i = \frac{\beta_iN_e}{\bar{N}_e}$, respectively. The rest of parameters remain the same. See Table 1 for further detail about parameters description.

### 3. Model analysis

Throughout this section, we first assume that $\theta_1(t) = \theta$ and $\delta_0 + \delta_1(t) = \delta_0$ are both constant in time. Then, the system (2) has a disease-free equilibrium

$$x^0 = \left(\frac{\bar{x} + \bar{\mu}_s + \delta_0 A}{K_2}, \frac{\bar{x} + \delta_0 A \delta_1 B}{K_1 K_2}, \frac{\bar{x} + \bar{\mu}_s + \delta_0 A \delta_1 B}{K_1 K_2}, \frac{\bar{v}_1}{\mu_1}, \frac{\bar{v}_2}{\mu_2} \right)$$

where $K_1 = \frac{(\bar{x} + \bar{\mu}_s + \delta_0)(\bar{x} + \bar{\mu}_s + \delta_10) - \delta_10}{\mu_1(\bar{x} + \bar{\mu}_s + \delta_10)}$ and $K_2 = \frac{\mu_1(\bar{x} + \bar{\mu}_s + \delta_10)}{\mu_2(\bar{x} + \bar{\mu}_s + \delta_10)}$.

In the case of no treatment and no age class division, i.e. $\theta_1 = \theta_2 = 0, \delta_0 = \delta_2 = 0, x_2 = x_3 = x_4 = x_5 = x_7 = x_8 = 0$, the system (2) is reduced to a standard dengue transmission model [4] with the known basic reproductive ratio

$$R_{00} = \sqrt{\frac{b_{ns} N_e}{\mu_s(\bar{x} + \gamma) N_v}}$$

The basic reproductive ratio $R_{00}$ represents the expected numbers of secondary cases produced by a typical infected individual during its entire period of infectiousness in a completely susceptible population (see the detail in [6,7]). The parameter $(R_{00})^2$ is in fact a measure of two-stage infection as follows. A single infected mosquito successfully infects $b_{ns}N_e$ humans per unit time during their infection period $\frac{1}{\mu_v}$. Each infected human then infects $\frac{b_{ns}N_e}{\mu_v}$
Table 1
Variables and parameters used in system (2) and their description. All variables and parameters are assumed to be non-negative.

<table>
<thead>
<tr>
<th>Variable/parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1(t), x_2(t)$</td>
<td>Untreated/treated susceptible children</td>
<td>$x_1(t), x_2(t) \geq 0$</td>
</tr>
<tr>
<td>$x_3(t), x_4(t)$</td>
<td>Untreated/treated susceptible adults</td>
<td>$x_3(t), x_4(t) \geq 0$</td>
</tr>
<tr>
<td>$x_5(t), x_6(t)$</td>
<td>Untreated/treated infected and infectious children</td>
<td>$x_5(t), x_6(t) \geq 0$</td>
</tr>
<tr>
<td>$x_7(t), x_8(t)$</td>
<td>Untreated/treated infected and infectious adults</td>
<td>$x_7(t), x_8(t) \geq 0$</td>
</tr>
<tr>
<td>$x_9(t)$</td>
<td>Recovered and immune humans</td>
<td>$x_9(t) \geq 0$</td>
</tr>
<tr>
<td>$p_1(t), p_2(t)$</td>
<td>Susceptible/infectious vectors</td>
<td>$p_1(t), p_2(t) \geq 0$</td>
</tr>
<tr>
<td>$A, B$</td>
<td>Human/vector recruitment rate</td>
<td>$\frac{\alpha}{N_1 N_3}$ and $\frac{\delta}{N_3}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Rate of transition from children to adults</td>
<td>$\frac{\alpha}{N_2}$</td>
</tr>
<tr>
<td>$\theta_1(t), \theta_2(t)$</td>
<td>Rate of treated children/adults</td>
<td>$[0, 1]$</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>Treatment frequency</td>
<td>1</td>
</tr>
<tr>
<td>$\delta_1(t), \delta_2(t)$</td>
<td>Drop-out rate of treated children/adults</td>
<td>$[0, 1]$</td>
</tr>
<tr>
<td>$\mu_l, \mu_r$</td>
<td>Natural death rate of humans/vectors</td>
<td>$\frac{\mu_l}{N_1} + \frac{\mu_r}{N_2}$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Human recovery rate</td>
<td>1</td>
</tr>
<tr>
<td>$\beta_l, \beta_r$</td>
<td>Transmission rate in humans/vectors</td>
<td>0.1 and 0.1</td>
</tr>
<tr>
<td>$b$</td>
<td>Mosquito biting rate</td>
<td>1</td>
</tr>
</tbody>
</table>

The element $G_{jk}$ represents the number of new infections in compartment $j$ being generated by a single infective from compartment $k$. Note that for $j$ and $k$ from 1 to 5 represents $x_1, x_2, x_3, x_4$, and $x_5$ respectively. For an example, element $G_{12}$ represent that every single infective person from compartment $x_2$ can infect $\frac{\mu_l}{N_1 N_3 + \mu_r}$ person in compartment $x_4$. The basic reproductive ratio $R_0$ of the system (2) is given by the spectral radius of $G$, i.e. $R_0$ is the largest eigenvalue of $G$. From the Perron theorem [13], the largest eigenvalue of $G$ is real and positive. Moreover, the characteristic polynomial of $G$ is given by

$$
\chi_G(\lambda) = \lambda^5 - a_3 \lambda^3 + a_2 \lambda^2 + a_1 \lambda - a_0,
$$

where the coefficients $a_i$ are positive and given by

$$
a_3 = \frac{\alpha}{N_1 N_4}, \quad a_2 = \frac{\zeta b B}{N_1 N_4}, \quad a_1 = \frac{\tilde{B} \left( \delta_2 \theta_2 N_2 + \tau \delta_1 \theta_1 N_1 + \delta_1 \theta_1 \theta_2 N_2 \right)}{N_1 N_2 N_1 N_4}, \quad a_0 = \frac{\zeta \delta_1 \theta_2 B}{N_1 N_2 N_1 N_4}.
$$

The basic reproductive ratio $R_0$ which satisfies $\chi_G(R_0) = 0$ may not be obtained analytically. We find that the disease free equilibrium is stable if and only if $R_0 < 1$ [6], or equivalently $\chi_G(R_0) > 0$. Before the treatment is given to the population ($\theta_1 = \theta_2 = \delta_1 = \delta_2 = 0$) and if there is no age class division ($\alpha = 0$), $R_0$ reduces to $R_{00}$.

The sensitivities of the parameters that affect $R_0$ are shown in Figs. 2 and 3 (see the parameters value in Table 2). It can be seen, that a relatively huge effort is required to reduce $R_0$, which is not feasible in practice due to the limited budget. For instance, in Fig. 2, with a constant rate of $\delta_1$ in $2.5 \times 10^{-7}$, larger $\delta_1$ will reduce $R_0$ much better. As a consequence, the budget is quite large because the drop-out rate ($\delta_1, \delta_2$) is quite small only $10^{-7}$. For daily application of control treatment, the rates of treatment and drop-out will depend on time. In this case, the basic reproductive ratio does not exist. Related to the optimal control analysis, it is shown in the next section that the result with optimal control is far more efficient (in terms of cost as well as outbreak reduction) in comparison with constant control.

In the next section we consider the case of the non-autonomous system (2), in which the epidemic condition is satisfied before the treatment, i.e. $R_0 > 1$. Due to further limitation of budget and resources the epidemic may not be removed within the admissible control parameters. In this case, the optimal treatment has to be designed such that the total number of the infected individuals is kept as small as possible within admissible control parameters.

4. Optimal control problem

Given the disease model (2) we want to design the treatment rates $\theta_1(t), \theta_2(t)$ and the drop-out rates $\delta_1(t), \delta_2(t)$ such that we minimize the number of the infected individuals. Obviously, this task can always be achieved by imposing extremely high treatment rates and minimal drop-out rates. However, high treatment rates...
The costs of the treatment use are more-or-less proportional to i.e.

have to choose negative weights \( b \) on the hazards, i.e. the drop-out-rates, the situation is just the other way:

The weighting parameters \( \omega_{a,j} \) and \( \omega_{b,j} \) are used for the state variables \( x \) and the control variables \( u = (\theta_1(t), \theta_2(t), \theta_3(t), \theta_4(t)) \), respectively. Since we are mainly interested in minimizing the number of the infected humans, we may set \( \omega_{a,i} > 0 \) for \( i = 1, \ldots, 8 \). The number of susceptible humans, i.e. \( x_1, \ldots, x_8 \) as well as the number of recovered individuals \( x_9 \) and mosquito population \( v_1 \) and \( v_2 \) is not taken into account, hence we choose the weights \( \omega_{a,9} = 0 \) for those compartments. On the other hand we want to minimize the costs for our treatment and campaign. The costs of the treatment use are more-or-less proportional to the treatment rates \( u_1 \) and \( u_2 \), hence we choose the weights \( \omega_{b,1}, \omega_{b,2} > 0 \), as usual. To the second set \( u_3, u_4 \) of control variables, i.e. the drop-out-rates, the situation is just the other way round: small drop-out rates require high costs and hence we have to choose negative weights \( \omega_{b,3}, \omega_{b,4} \), if we aim to minimize the costs.

Now, the optimal control task reads as

\[
\min_u J(x, u) \quad \text{s.t.} \quad P(x, u) = 0, \tag{9}
\]

where \( P(x, u) = 0 \) is a short hand notation for the state system (2), i.e. \( \dot{x} = M(t, x, u), x(0) = x_0 \). To derive the optimality conditions, we introduce the Lagrangian

\[
L(x, u, z) = J(x, u) + P(x, u, z). \tag{10}
\]

The duality product \( \langle P(x, u), z \rangle \) is induced by the \( L^2 \)-inner product

\[
\langle P(x, u), z \rangle = \frac{1}{T} \int_0^T (\dot{x} - M(x, u)) z \, dt = x(T) \cdot z(T) - x_0 \cdot z(0) + \int_0^T x \cdot \dot{z} - M(x, u) z \, dt.
\]

The first order optimality condition states that an optimum of the problem (9) is a stationary point of \( L \), see \([21,12]\). Hence \( \partial_u L = \partial_x L = \partial_z L = 0 \). Taking the derivative with respect to the adjoint variable \( z \) yields the state system \( \dot{x} = M(x, u) \).

Next, taking the derivative with respect to the state variables \( x \) yields the adjoint system

\[
\dot{z} = -\partial_x M(x, u) z + \partial_z J(x, u) z(T) = 0. \tag{11}
\]

In the case of the disease model (2), we obtain the following set of equations

\[
\begin{align}
\dot{z}_1 &= \mu_x z_1 + \alpha (z_1 - z_3) + b_x x_1 (z_1 - z_3) + \theta_1 (z_1 - z_2), \\
\dot{z}_2 &= \mu_x z_2 + \alpha (z_2 - z_4) + (\theta_0 + \theta_1)(z_2 - z_1), \\
\dot{z}_3 &= \mu_x z_3 + b_x x_1 (z_1 - z_3) + \theta_2 (z_2 - z_3), \\
\dot{z}_4 &= \mu_x z_4 + (\theta_0 + \theta_2)(z_4 - z_1), \\
\dot{z}_5 &= \mu_x z_5 + \alpha (z_5 - z_7) + \gamma (z_5 - z_9) + b_x x_10 (z_{10} - z_{11}), \\
\dot{z}_6 &= \mu_x z_6 + \alpha (z_6 - z_8) + \gamma (z_6 - z_9) + (\theta_0 + \theta_1)(z_6 - z_5) + \omega_{b,6} x_6, \\
\dot{z}_7 &= \mu_x z_7 + \gamma (z_7 - z_9) + b_x x_10 (z_{10} - z_{11}) + \theta_2 (z_7 - z_8) + \omega_{b,7} x_7, \\
\dot{z}_8 &= \mu_x z_8 + \gamma (z_8 - z_9) + (\theta_0 + \theta_2)(z_8 - z_7) + \omega_{b,8} x_8, \\
\dot{z}_9 &= \mu_x z_9, \\
\dot{z}_{10} &= \mu_x z_{10} + b_x (x_5 + x_7) (z_{10} - z_{11}), \\
\dot{z}_{11} &= \mu_x z_{11} + b_x (x_5 (z_1 - z_3) + x_7 (z_2 - z_4)).
\end{align} \tag{12}
\]

All of these equations are subject to the condition \( z(T) = 0 \).

Finally, taking the derivative with respect to the control variable \( u \) yields the gradient equation

\[
\partial_u J(x, u) \cdot u = 0. \tag{13}
\]

In the setting of the disease model (2) this reads as

\[
\begin{align}
\omega_{b,1} u_1 + x_1 (z_1 - z_2) + x_5 (z_5 - z_6) &= 0, \\
\omega_{b,2} u_2 + x_3 (z_1 - z_4) + x_7 (z_7 - z_8) &= 0, \\
\omega_{b,3} u_3 - x_5 (z_1 - z_3) - x_6 (z_5 - z_6) &= 0, \\
\omega_{b,4} u_4 - x_4 (z_1 - z_4) - x_8 (z_7 - z_8) &= 0.
\end{align} \tag{14}
\]
Solving these equations for the vector \(-u\), we obtain the direction of the negative gradient \(D_J(x, u)\) at the point \((x, u)\).

With these ingredients at hand, we are now in position to setup a gradient method for solving the minimization problem.

1. Choose some initial guess \(u^0\) for the control.
2. Solve the state Eq. (2) to compute \(x\).
3. Compute the actual cost \(J\).
4. Solve the adjoint Eq. (12) to compute \(z\).
5. Solve the gradient Eq. (14) to compute the negative gradient \(D_J\).
6. Solve an approximate line search to find \(s^*\) where \(u = u + s \cdot D_J(x, u)\).
7. Set \(u = u + s^* \cdot D_J(x, u)\).
8. Go to 2, unless termination criteria are met.

This algorithm in pseudo-code requires some additional comments.

Steps 2 and 4 require the solution of a system of ordinary differential equations by any suitable numerical scheme. The state equation has to be solved forward in time, integrating from 0 to \(T\) while the adjoint equation requires backward integration from \(T\) down to 0.

Step 6 is one of the crucial steps within the algorithm. Of course, performing an exact line search, i.e. determining \(s\) such that

\[
\min \{J(x(u_1), u_1)\} = J(x(u_t), u_t)
\]

would be the optimal choice regarding accuracy. However, determining \(s\) usually requires many evaluations of \(J\), i.e. one has to solve the state equation quite a number of times. To avoid the time consuming line search one has to resort to some heuristics. The simplest one is using a fixed stepsize \(s_0\) in the direction of the negative gradient. Being very simple, this method faces convergence problems. Another, quite popular, choice is the Armijo-rule [14], starting with a stepsize \(s_0\), one tries the sequence \(\beta^j s_0\) for some \(0 < \beta < 1\) of step sizes, until the cost functional is reduced for the first time. The reasoning behind this heuristics is the following. The overall aim is to reduce the cost functional in each step to avoid oscillations. Knowing, that going a small step in the direction of negative gradient will always lead to a decrease of the cost functional, we have to reduce step sizes that are too large and lead to an increase of \(J\). More advanced methods for determining a suitable value \(s^*\) can be thought of, including local polynomial approximations of the cost functional, see [14] for further details.

5. Numerical results

In this section we present some numerical simulations to the optimal control problem. Let \(\omega_{k,i}, i = 1, 2, \ldots, 9\) be the weight for social cost in human (hospitalization), \(\omega_{k,i}, i = 10, 11\) for mosquito and \(\omega_{k,j}, j = 1, 2\) be the weight cost in treatment rates for children and adults. For \(j = 3, 4\), we define \(\omega_{k,j}\) as the weight for the government campaign cost on the importance of repellent for children and adults, respectively. The choice of these weights reflects the different scales of the costs for counselling and treatment. Here we used \(\omega_{ij} = (1, 2, -7, -10)\) for weight in control parameters and the following weights in Table 3 for state variables in the cost functional (8). The weight control treatment parameter for adults \(\omega_{h,2}\) is twice as large as that for children \(\omega_{h,1}\). This is related to the fact that children are much easier to target for treatment than adults.

Numerical simulations of the optimal control problem are carried out for two scenarios, namely the case of prevention and the case of epidemic reduction. The case of prevention occurs at the early state of transmission, i.e. when the number of infected people is still relatively small before the start of the treatment. In the case of epidemic reduction, the treatment starts during the outbreak period, i.e. when the number of infected people is significantly

### Table 3

<table>
<thead>
<tr>
<th>Weight parameters</th>
<th>Compartment</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\omega_{k,i})</td>
<td>(x_1)</td>
</tr>
<tr>
<td>(0)</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 4

<table>
<thead>
<tr>
<th>Initial conditions for both scenario.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>Prevention</td>
</tr>
<tr>
<td>Reduction</td>
</tr>
</tbody>
</table>

![Fig. 4. Dynamics of the infected compartment \((x_1 + x_6 + x_7 + x_8)\) in the prevention scenario.](image)

![Fig. 5. Dynamics of the susceptible compartment \((x_1 + x_2 + x_3 + x_4)\) in the prevention scenario.](image)
As an initial condition, we assume that in the field the ratio between the mosquito and human population is $\frac{1}{2}.$ With this ratio, the corresponding basic reproductive ratio is about $1.414$ before the start of the treatment. The initial conditions for both scenarios are shown in Table 4.

In the case of prevention, the main goal of the treatment is to prevent the occurrence of an epidemic in the population or to reduce the intensity of the epidemic if it already started. Hence we want to suppress the outbreaks as much as possible within the given budget constraints. The dimensionless value of the cost functional (8) for one unit period of treatment is $4829.$ Simulation results for total infected compartment and total susceptible compartment in the prevention case are shown in Figs. 4 and 5, respectively. It can be seen in Fig. 7, with the related optimal control function shown in Fig. 6, the total infected population at the peak of the outbreak could be suppressed to a level two times smaller than without treatment along with delaying of the outbreak.

For the case of epidemic reduction, the optimal controls are shown in Fig. 9. In the epidemic reduction scenario, the treatment rate of the population is higher than in the prevention case to prevent the return of the outbreak. The achieved reduction of the total number of infected persons and the susceptible population as a consequence of the high rate of the treatment is shown in Figs. 7 and 8. In this scenario, the total cost for one unit period of treatment equals $90,440$ being almost twenty times greater than the minimal cost in the prevention scenario. Hence, prevention of the disease should be preferred to just later reduction of the epidemic.

Neglecting the age structure, i.e. $x = 0,$ the mathematical model (2) is reduced to a 7-dimensional system. The corresponding weight cost for control parameters $\theta_1$ and $\delta_1$ are $2$ and $-10,$ respectively, and weight parameters for state variables are given in Table 5. The state variables $x_3, x_4, x_7, x_8$ and the control variables $\theta_2, \delta_2$ are eliminated since there is no age class in the model.

The comparison between the dynamics of the two models (with and without age class) for the prevention and epidemic reduction scenario are shown in Fig. 10, left and right, respectively. The corresponding actual control function for the prevention case is shown in Fig. 11. It can be seen that although the dynamic of the infected compartment is only slightly different between the models with and without age class, the total cost for each case is significantly different, i.e. $4829$ and $5528,$ respectively. Similarly, in the epidemic reduction case (see Fig. 12),
the corresponding costs are 90,440 and 98,906 respectively. The significant difference in the cost value here is due to the absence of a children’s compartment in the case with no age class, with the consequence that everybody is treated using the adult rate (which is more expensive).

Table 5
Weight parameters when $x = 0.$

<table>
<thead>
<tr>
<th>Weight parameters</th>
<th>x₁</th>
<th>x₂</th>
<th>x₃</th>
<th>x₄</th>
<th>x₅</th>
<th>x₆</th>
<th>x₇</th>
<th>x₈</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{k,j}$</td>
<td>0</td>
<td>0</td>
<td>400</td>
<td>400</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
6. Conclusion

We have presented a two-age class model for Dengue transmission with mosquito repellent as a control mechanism. The progress of the outbreak of the disease is influenced by several control parameters, namely the treatment and the drop-out rates for both children and adults. The basic reproductive ratio before the treatment is obtained implicitly as the positive root of the fifth order characteristic polynomial of the next generation matrix. An optimal control problem has been formulated for the above Dengue transmission model. We have derived a gradient algorithm to solve the optimization problem. For two typical scenarios, the prevention of an outbreak and the reduction of an existing epidemic, we have carried out numerical simulations to compute the optimal treatment and drop-out rates. The significance of the age structure is indicated in the calculation of the optimal cost. The higher cost value in the case with no age structure is simply due to the use of adults unit treatment cost for all persons.

With a limited budget, it is much better to apply the treatment well before the occurrence of the outbreak. This can be done if it is known that an epidemic of Dengue will take place, assuming that the ratio between the mosquito and human population is known and hence the basic reproductive ratio can be estimated. This work could be considered as a first approach to finding a simple treatment for a complicated Dengue transmission problem. Further research could study integrated treatments for human as well as for vectors. In a more diverse population, the impact of spatial heterogeneity may also be taken into account.

Acknowledgements

The authors would like to thank the referees for their valuable comments and suggestions. Parts of the research are funded by the International Research Grant of the Indonesian Directorate General for Higher Education and the German Academic Exchange Service (DAAD).

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