A mathematical model for the Andean Tiwanaku civilization collapse: Climate variations

J.C. Flores, Mauro Bologna, Deterlino Urzagasti

Abstract

We propose a mathematical nonlinear model for the Tiwanaku civilization collapse based on the assumption, supported by archeological data, that a drought caused a lack of the main resource, water. We evaluate the parameter of our model using archaeological data. According to our numerical simulation the population core should have decreased from 45,000 to 2,000 inhabitants due to lake surface contraction.

1. Introduction: the Tiwanaku civilization

Why do some societies collapse? The answer to this is varied and complex. Tainter (2007) points this out in his book. There are two major opposing responses: on one side is the belief in internal causes, that social conflict destroys the established order; on the other side is the concept of an uncontrollable external input, such as severe climatological change. In a realistic case it is reasonable to assume that both aspects are present. Tainter's book points out that in principle, a complex society could manage any external input, or stated otherwise, a complex society could eventually adapt to the external change. The collapse, if it occurs, has an internal dynamic due to poor management strategies.

Diamond (2005) presents a different concept. According to him, among several causes, one could be that an external input could directly produce a collapse. We propose that the latter concept, rather than the former, played a more important role in the collapse of a pre-Columbian Andean civilization, the Tiwanaku. We stress the fact that in literature are present rather sophisticated modelling approaches related to the collapse of Tiwanaku as studied by Griffin and Stanish (2007). In that work, an agent-based model is considered and an interdisciplinary aspect is studied. In our study we limit ourselves to a macro-dynamic based on exploitation of the natural resources.

The Tiwanaku civilization started around 300 B.C. reaching a stable number of inhabitants around 1000 A.D. The Tiwanaku settlement was situated near the southern part of the Titikaka Lake which was known either as Minor Lake or Winaymarka lake (see Fig. 1). According to Binford et al. (1997), a prolonged period of drought (1100–1400 A.D.) caused the decline of agricultural production and subsequently field abandonment, and this led to the society's collapse. The Tiwanaku's demise, according to Binford et al. (1997) and Delclaux et al. (2007), was due to the lack of water draining into their raised-fields. In fact, the epicenter of the Tiwanaku civilization was around Minor Lake, a lake that was altered by climatological changes (see Fig. 1). Another plausible scenario suggests that the decay of water levels may be due to water conflicts between the Wari and the Tiwanaku (Williams, 2002). Generally speaking, climate variations played a major role in other archaeological cases. Jimenez-Espejo et al. (2007) and Finlayson (2004) conjecture that Neanderthal extinction was due mainly by the last glaciation (50,000–12,000 years ago). In this case the alternative conjecture for extinction was competition with Modern Humans (Banks et al., 2008; Flores, 2011).

2. Model for the Tiwanaku collapse

In our mathematical model we consider two basic dynamic variables; the number of inhabitants of the extinct Tiwanaku society, and the main resource of water. We simplify the problem by assuming, supported by archeological data, that a drought caused a lack of the main resource, water. We evaluate the parameter of our model using archaeological data. According to our numerical simulation the population core should have decreased from 45,000 to 2,000 inhabitants due to lake surface contraction.
is the extension of Minor Lake, denoted by $S$, which is directly related to the quantity of water. We will assume that the collapse of the Tiwanaku civilization was due to the contraction of the surface related to the aforementioned drought and to their agricultural needs. The other dynamic variable of the system, denoted by $N$, is the number of inhabitants living in the city around Minor Lake. To determine the number of inhabitants $N$, we will consider a logistic equation (Murray, 2002) with a growth rate of $r=0.01$ yr$^{-1}$. This value was obtained by Fort and Mendez (1999) and used in other models of collapsed cultures (Eastern Island and Maya of Copan, Bologna and Flores, 2008). The carrying capacity of the system will be assumed depending on accessibility to the water. The assumption is that the carrying capacity is proportional to a linear function of the extension of the lake, or more precisely, the exploited surface $S$ of the lake by the inhabitants. This hypothesis is in agreement with the model of Eastern Island in Bologna and Flores (2008) although we must point out that other valid possibilities could be considered. In reference to this paper, a valid alternative is represented by considering as a resource the volume of water instead of the surface of the order of 7000 km$^2$; and Minor Lake, or Winaymarka lake, with a modern maximum depth of 41 m and a surface of the order of 7000 km$^2$; and Minor Lake, or Winaymarka lake, with a modern maximum depth of 41 m and a surface of 1400 km$^2$. These basins are connected by a thin link (Estrecho de Tiquina) (Fig. 1), a detailed knowledge of it is difficult to obtain. Despite this fact, and with respect to the surface variation, the available archaeological data allows a rough estimation of this flux change. In Eq. (2), $\alpha$ is the evaporation rate parameter, evaluated approximately as $\alpha=1.6 \times 10^{-3}$ km/yr (Delcaux et al., 2007), $S$ is the total surface of Minor Lake and $d_o$ is the rate parameter representing the water that was drained due to the needs of the population.

The system of coupled differential equations described by Eqs. (1) and (2) has two equilibrium points. The unstable (saddle point) given by

$$N_{st} = 0, \quad S = \frac{f}{\alpha}.$$  

which corresponds to the absence of human activity around the lake and the stable point

$$N_e = R_o S, \quad S = \frac{f}{\alpha + d_o R_o}.$$  

From Eq. (4) we deduce a relation between the number of inhabitants and the area of Minor Lake. Indeed, it is closely related to the archaeological data and the estimation found in this paper. If, due to climatological variations, we assume a change in the total water flux $f \rightarrow f'$, then the lake surface must also change $S \rightarrow S'$. Under these circumstances, the number of individuals will be adjusted to the constriction of the lake $N_a \rightarrow N_{st}$. Explicitly, from the first of Eq. (4) we have

$$\frac{N_a}{N_e} = \frac{S'}{S}.$$  

This is quite an important relation since archaeological knowledge about the surface variations becomes directly related to the population change. In the same way, the total flux variation $(f \rightarrow f')$ is related to the surface change (see Eq. (4)) by $f'S = fS'$.  

### 3. Evaluation from archaeological data: prediction of inhabitants collapse

Titikaka lake is composed of two basins (Fig. 1): Major Lake, with a modern approximative maximum depth of 283 m and a surface of the order of 7000 km$^2$; and Minor Lake, or Winaymarka lake, with a modern maximum depth of 41 m and a surface of 1400 km$^2$. These basins are connected by a thin link (Estrecho de Tiquina). In our model, and from an agricultural point of view, we assume that the inhabitants of Tiwanaku were essentially related with Minor Lake due to the closeness of their population core area with its basin, while the level of the entire lake was governed by a physical water balance. The lake level and the climatological situation before the drought were very similar to modern days (Abbott et al., 1997; Binford et al., 1997). The estimated minimum values at the epoch of collapse are 5800 km$^2$ for Major Lake and 60 km$^2$ for Minor Lake (Binford et al., 1997).

The number of persons before the collapse is widely debated. Nevertheless, according to Tainter (2007), the maximal population of Tiwanaku’s city core was around 45 000 inhabitants. The evaluation of the parameter $R_o$ comes directly from Eq. (4), $R_o = N/S = 45 000/1400 \sim 32$ km$^2$. From Eq. (5), and using the aforementioned data for the surface variations, we can evaluate the number of individuals after the collapse, as $N' = (60/1400) \times 45 000 \sim 2000$ inhabitants. This is the main result of this work.
or increasing the time to arrive at the saturation point. The final value is $N_s \approx 2000$.

Fig. 2. Number of inhabitants as a function of time. The time origin is taken at 300 B.C. where we assumed an initial number of individuals of the order of 100. This initial value affects the first stages of the time evolution of $N(t)$ simply decreasing or increasing the time to arrive at the saturation point. The final value is $N_s \approx 2000$.

Fig. 3. Minor Lake surface as a function of time. The time origin is taken at 300 B.C. The final value is $S_{\text{fin}} = 60 \text{ km}^2$.

To continue the evaluation of the parameters, we shall assume that the drained water was not relevant in the collapse. This is equivalent to setting $d_o = 0$ in Eq. (4). Then, we set for the total flux after collapse $f = 2S = 1.6 \times 10^{-3} \times 60 \approx 9.6 \times 10^{-2} \text{ km}^3/\text{yr}$ and, for the total flux before collapse $f = 2S = 1.626 \times 10^{-3} \times 1400 \approx 2.2 \text{ km}^3/\text{yr}$. We stress that, in both mathematical archaeology and mathematical biology the parameters estimation is extremely difficult. In this sense there are several important studies about the estimation of the parameter values (Abbott et al., 1997; Brenner et al., 2001; Dearing et al., 2006; Erickson, 1999; Ortloff and Kolata, 1993; deMenocal, 2001).

4. Numerical calculations

In this section we present the results obtained by numerically solving Eqs. (1) and (2) (see Figs. 2 and 3). We have considered archaeological data discussed in the previous section. Fig. 2 shows the numerical simulation for the number of individuals $N$ as a function of time. As shown in this figure, there is a significant growth of the Tiwanaku population between 300 B.C. and 1000 A.D., reaching a stable saturation point of approximately 45 000 individuals. Then, starting from 1100 A.D. up to 1400 A.D., there are operative climatological changes and the population adjusts to the new carrying capacity. According to our calculations, this is of the order of 2000 individuals (see previous section).

To perform the numerical calculations Eq. (2) requires an explicit dependence between volume $V$ and surface $S$. We sketched Minor Lake as a segment of a sphere and Eq. (2) has been rewritten as

$$\frac{S}{2\pi r_{W}} \frac{dS}{dt} = f - 2S.$$  

(6)

Since the depth of the segment of the sphere $h \approx 40 \text{ m}$ is much smaller than the radius of the sphere, $r_{W} \approx 5 \times 10^3 \text{ km}$, volume $V$ can be expressed as $V \approx \frac{S^2}{4\pi r}$. Note that the ratio $h/r_{W} \approx 10^{-5}$ is a very small number, so other approximations made with other regular surfaces would differ little from a spherical basin. For the sake of simplicity we schematized the function $f$ as the step function $h(t)$, namely $f \approx 2.30(1100-t)+0.1(t-1100)$. The use of smoother functions would show less dramatic change in $N(t)$ and $S(t)$ for the period from 1100 A.D. to 1500 A.D., but the final result would still be the same.

5. Conclusions

In this paper we considered the evolution of the number of individuals of the Tiwanaku civilization from its growth to its demise. The number of individuals $N(t)$ has been evaluated as a dynamic variable coupled nonlinearly with the inhabitants’ main natural resource, the water of Minor Lake Eqs. (1) and (2). Climatological effects were assumed to cause a severe decline of the surface area of Minor Lake from 1400 km$^2$ to 60 km$^2$, reducing the number of individuals at Tiwanaku’s epicenter. According to our model the inhabitants ranged from 45 000 to 2000.

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